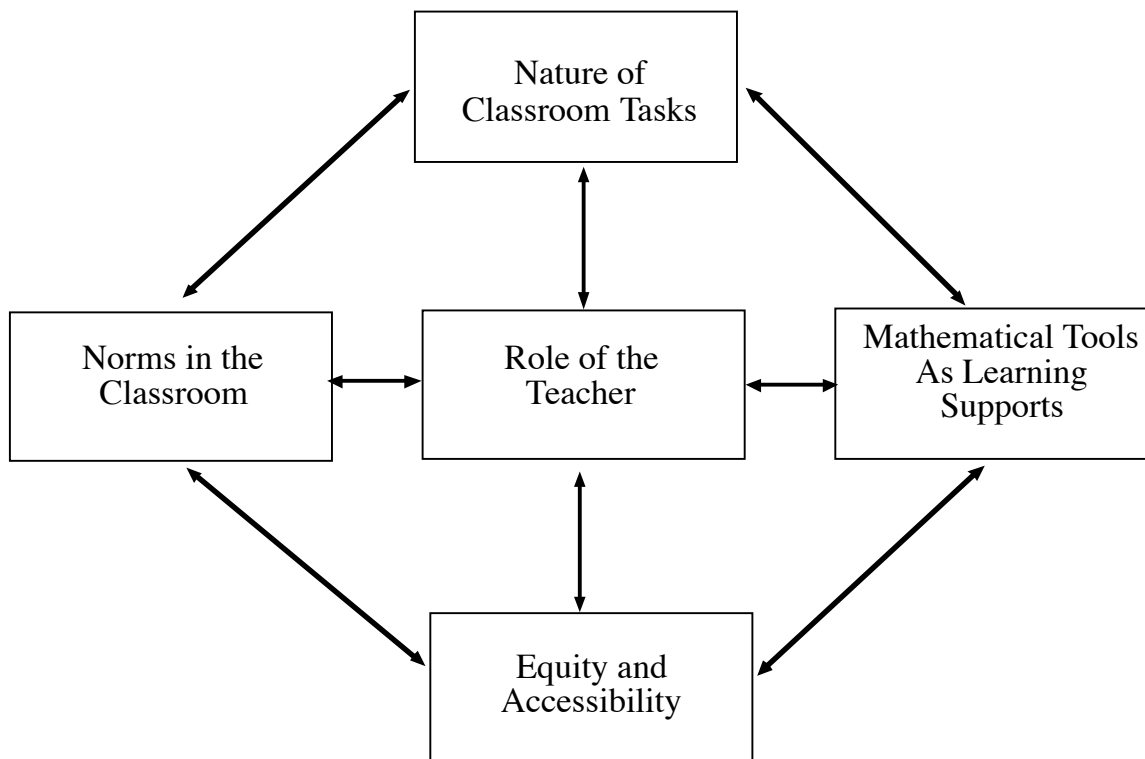Problem solving . . . can serve as a vehicle for learning new mathematical ideas and skills. . . . A problem-centered approach to teaching mathematics uses interesting and well-selected problems to launch mathematical lessons and engage students. In this way, new ideas, techniques, and mathematical relationships emerge and become the focus of discussion. Good problems can inspire the exploration of important mathematical ideas, nurture persistence, and reinforce the need to understand and use various strategies, mathematical properties, and relationships.

(*Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics 2000, p. 182)

System of Mathematics Classroom Instruction

J. Hiebert, et al.

Nature of Classroom Tasks

Norms in the Classroom	Role of the Teacher	Mathematical Tools As Learning Supports

Equity and Accessibility

Selecting Classroom Tasks - Basic Questions
R. Marcus & J. T. Fey

Will working on the tasks foster students' understanding of important mathematical ideas and techniques?

- Will the selected tasks be engaging and problematic, yet accessible, for many students in the target classes?
- Will work on the tasks help students develop their mathematical thinking—their ability and disposition to explore, to conjecture, to prove, to represent, and to communicate their understanding?
- Will the collection of tasks in a curriculum build coherent understanding and connections among important mathematical topics?

Some Questions That Promote Understanding
D. A. Grouws

How did you decide on a solution method to try?

How did you solve the problem?

Did anyone solve it in a different way?

How would you compare these solution methods?

Which of the solution methods do you like best? Why?

Can you tell me how you solved the problem without saying the answer?

Does this remind you of any other problems you have solved?

How can we change the problem to get another interesting problem?

What mistakes do you think some students might make in solving this problem?

Interesting Variations on a Basic Problem
Goldenberg & Walter

Find the mean of 7, 4, 7, 6, 3, 8, and 7.

1. What if only five of the seven data are given? Can we determine the missing data if we know the mean of the original seven?

2. What if we compute the mean of each possible combination of only five of the given seven numbers? (How many such combinations are possible?) What could we learn from, say, a histogram of those means?

3. What if the original seven numbers are sampled from a population consisting of eight numbers? What might we reasonably infer about the eighth number? Do ideas from problem 2 help answer that question?

4. What if we know the mean but none of the data? What, if anything, could we say about the data? What possible sets of data would fit?

5. What if the numbers are all different? Can we find seven different integers whose mean is 6? What if we also had to make the median 7 or the range 5?

6. What if the highest number is 18, not 8? Which of mean, mode, median, or range changes?

7. What if two sets of data are given instead of one? Suppose the second set is 2, 6, 7, 9. Can we calculate the mean of the combined sets (all eleven numbers) by combining the means of the two sets in some way? Would averaging the means do that job?

8. What if we know only that the means of two sets of numbers are 7 and 10? What, if anything, can we say about the mean of the combined set?

9. More generally, suppose that the set has n members and that the mean is m. What can we say about the numbers?

Teaching Mathematics through Problem Solving: Research Perspectives

M. K. Stein, J. Boaler, & E. A. Silver

The research on TMTPS and on curricula designed to support it suggests both the feasibility and efficacy of this approach.

When TMTPS is implemented effectively, students (compared to those taught traditionally) are likely to better understand mathematical concepts, to be willing to tackle challenging problems, and to see themselves as capable of learning mathematics.

TMTPS is challenging and to do it well teachers need support, including good curriculum materials and strong professional development.

TMTPS can work with a wide range of students, but the level of student support required may differ depending on the students' mathematical background and interest.