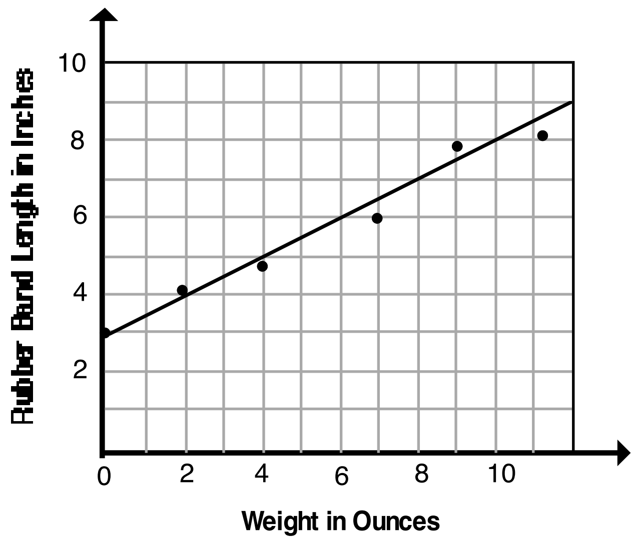
## 2 Linear Graphs, Tables, and Rules

When two variables are related in a pattern that is roughly linear, it is helpful to model the pattern with a straight line graph. The scatterplot below shows data from a test of a single rubber band cord in a bungee jump experiment. A linear graph has been drawn to model the pattern of change in length for increasing weight.




### Think About This Situation

The graph shows the overall pattern relating *weight* and bungee cord *length*. In a real bungee jump business, operators of the jump would have to quickly and accurately figure the stretched length for any jumper's weight.

(a). Based on the linear model (not the data points themselves), what pattern would you expect in a table of (*weight, length)* pairs of weights from 0 to 10 ounces?

(b). How long is this rubber band bungee cord with no weight attached, and how is that fact shown on the graph?

(c). How much does the rubber band bungee cord stretch for each ounce of weight added, and how is that shown on the graph?

## INVESTIGATION 2.1: Stretching Things Out

The bungee jump experiments involve one of the most common examples of linear models – the way that forces stretch rubber bands or coil springs. The more weight on the bungee cord, the longer it stretched. The same is true about coil springs.

An equation for a linear model gives the simplest and most useful summary of the relation between the variables. In the following activities you will explore the variety of patterns that can occur in linear equations as they model or rubber bands being stretched by increasing forces. As you complete the activities in this investigation, look for clues that will help you answer this basic question:

*How are patterns in linear graphs, tables, and equations related to each other?*

1. One key to the connection between graphs, tables, and equations for linear models is the **rate of change** in the two variables. Look back at the graph model on page 18.

   (a). If change in weight is represented by $\Delta$ **weight** (read "delta weight") and the matching change in rubber band length by $\Delta$ **length** (read "delta length"), the rate of change in length as a function of weight is given by $\frac{\Delta \textbf{length}}{\Delta \textbf{weight}}$.
   For example, the graph model passes through the points (0,3) and (10,8). The change in length between these points is (8 – 3) = 5; the change in weight is
   (10 – 0) = 10. Thus the rate of change is $\frac{5}{10} = \frac{1}{2}$.
   Find the rate of change for the same rubber band when the change in weight is:
   - 1 ounce
   - 2 ounces
   - 4 ounces

   (b). How is the rate of change shown in the linear graph?

   (c). The length of the rubber band with no weight is 3 inches. Use that fact and the rate of change to find the length of the stretched rubber band when the weight is:
   - 2 ounces
   - 4 ounces

- 10 ounces
- W ounces

(d). Write an equation using the words NOW and NEXT showing how the length of the stretched rubber band changes for each ounce of weight added.

(e). Using the letters L (for *length in inches*) and W (for *weight in ounces*), write an equation that shows how the two variables are related: L = ____.

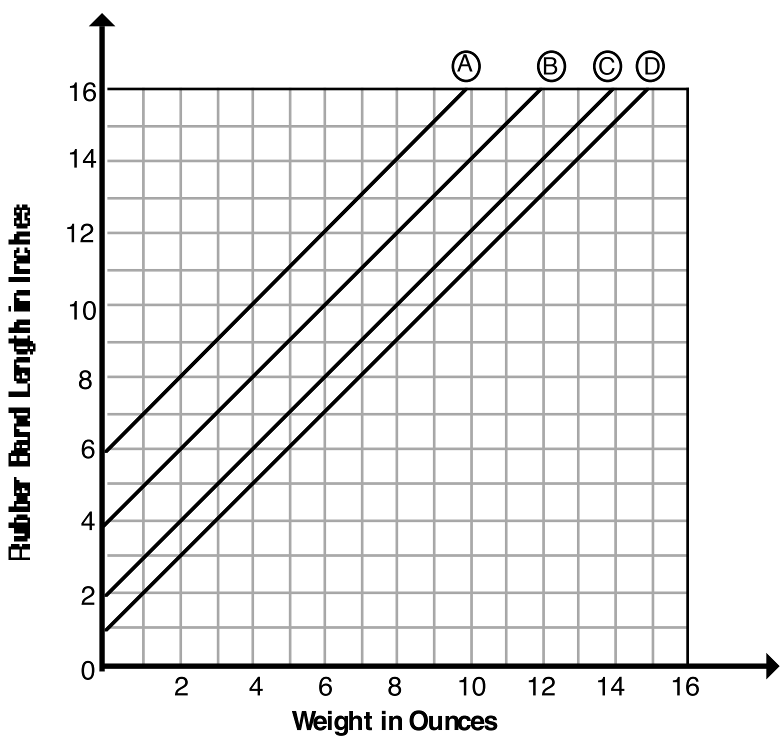
(f). How is the rate of change shown in the equations of parts (d) and (e)?

(g). How is the length of the rubber band with no weight attached shown in the equation for part (e)?

(h). Use your equations to predict stretch lengths for each of these weights:

- 4.5 ounces
- 17 ounces
- 0.25 ounces

2. The next diagram shows linear models from four rubber band experiments, all plotted on the same grid. What does the pattern of those graphs suggest about the similarities and differences in the experiments?



(a). Sharing the work among your group members, make four tables of (*weight*, *length*) pairs, one table for each linear model, for weights from 0 to 10 ounces.

(b). According to the tables, how long were the different rubber bands without any weight attached? How is that information shown on the graphs?

(c). Looking at data in the tables, estimate the rates of change in length for the four rubber bands as weight is added. How are those patterns shown on the graphs?

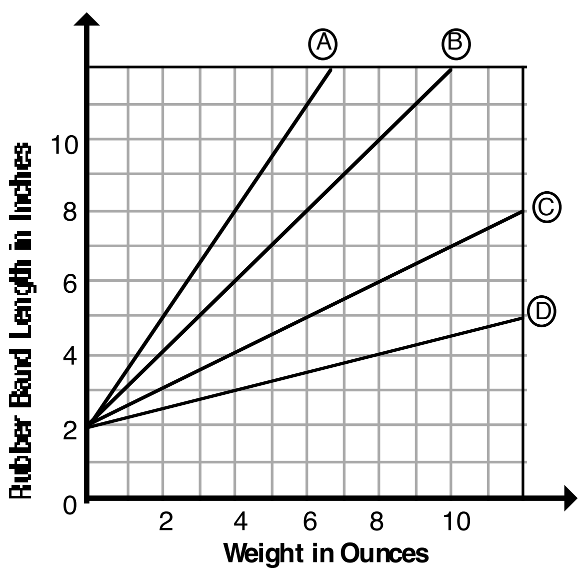
(d). Write an equation using NOW and NEXT showing how the rubber band length changes with each added ounce of weight in each case.

(e). Using the letters L (for *length in inches*) and W (for *weight in ounces*), write equations that show how the two variables are related in each case.

(f). How can you use an equation from part (e) to determine the length of a rubber band with no weight attached?

(g). How are the rates of change in length of the various rubber bands related? How is this fact shown in the equations?

3. The diagram below shows graphs of linear models from another set of four rubber band-weight experiments. What does the pattern of those graphs suggest about similarities and differences in the experiments?




(a). Sharing the work among your group members, make tables of (*weight*, *length*) pairs for each linear model A–D, for weights from 0 to 10 ounces.

(b). According to the tables, how long were the rubber bands without any weight attached? How is that information shown on the graphs?

(c). Looking at the data in the tables or the graphs, estimate the rates of change in length as weight is added. How can the rates be determined from the graph? From the table?

(d). For each linear model, write an equation using NOW and NEXT showing how the length changes as each ounce of weight is added.

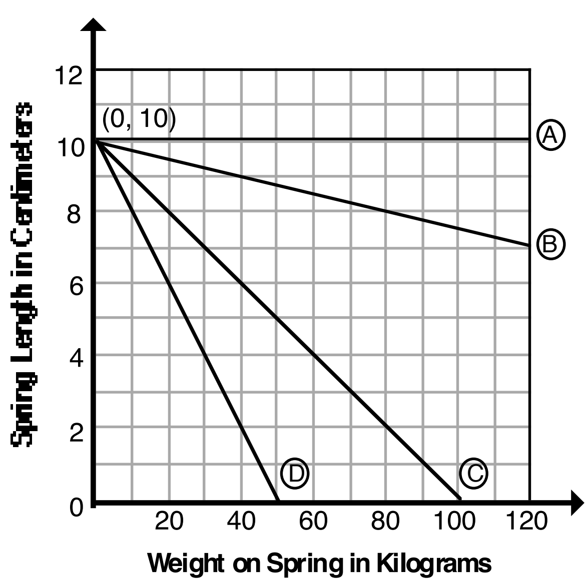
(e). Using the letters L (for *length in inches*) and W (for *weight in ounces*), write equations that show how the two variables are related in each case.

(f). What do the numbers in your equations tell you about the graphs?

(g). What differences in the rubber bands could cause the differences in graphs, tables, and equations that modeled the data from the experiments?

4. In some places where springs are used, the springs are made shorter by pressure from force or weights. For example, the springs in a bed or chair are pressed shorter when you sit on them. The springs in a scale are pressed shorter when you stand on the scale.

The diagram below shows graphs of four different linear relations. Those graphs model data from an experiment which tested bed springs. What does the pattern of those graphs suggest about the similarities and differences in the springs?



(a). Sharing the work with members of your group, make four tables of (*weight*, *length*) pairs, one table for each linear model, for weights starting at 0 kg.

(b). How long are the springs with no weight applied?

(c). What are the rates of change in spring length with increasing weight? How can you calculate these rates using points on the lines? Using pairs of values in the tables?

(d). Write equations using NOW and NEXT showing how the springs change length as each kilogram of weight is added.

(e). For each linear model, write an equation expressing L (*length in centimeters*) as a function of W (*weight in kilograms*).

(f). Which linear model might correspond to springs in an extra firm bed? In a medium firm bed? Explain your choices.

(g). For each of the tested springs, estimate the length of the spring with a weight of 30 kg. What do you prefer to use in making estimates – a graph, table, or equation?

5. Each linear equation below models a spring stretching or compressing experiment. Identify: A, the initial length of the spring; B, the rate of change of the length and; C, whether the experiment was designed to measure spring stretch or compression.

(a). L = 5 + 4W

(b). L = 1 + 2.3W

(c). L = 3 + -1.5W

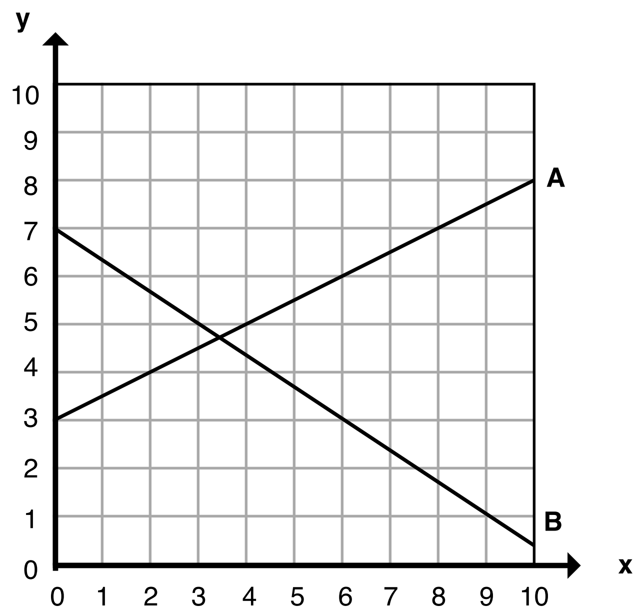
(d). L = 8 + -0.1W

It helps to think about weights and lengths of rubber bands or springs when studying a linear graph. But the connections between graphs, tables, and equations are the same for linear models relating any two variables *x* and *y*. By now you've probably noticed these key features of linear models and their graphs.

- Linear models always have a **constant rate of change**. That is $\frac{\Delta y}{\Delta x}$ is a constant.
- The constant rate of change can be seen in the **slope** of the linear graph. The **slope** is the direction and steepness of a walk along the graph from left to right.
- The ***y*-intercept** of the linear graph is the point where the graph intersects the *y*-axis.

*How could the connections between tables, graphs, and equations of linear models be used to locate the y-intercept and to measure the slope of its graph?*

6. Study the two linear models on this graph.




(a). Find the *y*-intercept of each graph. Then explain how those *y*-intercepts relate to:
- Tables of $(x, y)$ values for each graph.
- Equations relating NOW and NEXT for each graph.
- Equations relating $x$ and $y$ for each graph.

(b). Find the slope of each graph. Then explain how the slopes relate to:
- Tables of $(x, y)$ values for each graph.
- Equations relating NOW and NEXT for each graph.
- Equations relating $x$ and $y$ for each graph.

7. Look back over the examples of linear models for (*weight*, *length*) data from experiments with rubber bands and springs.

(a). How are linear graphs related when they have the same slope?

(b). How will linear graphs with the same *y*-intercept, but different slopes, be related to each other?

## Checkpoint

Linear models relating any two variables $x$ and $y$ can be represented using tables, graphs, or equations. Important features of a linear model can be seen in each representation.

(a). How can the rate of change in two variables be seen:

- In a table of $(x, y)$ values?
- In a linear graph?
- In an equation relating NOW and NEXT for the model?
- In an equation relating $x$ and $y$?

(b). How can the $y$-intercept be seen:

- In a table of $(x, y)$ values?
- In a linear graph?
- In an equation relating NOW and NEXT for the model?
- In an equation relating $x$ and $y$?

Be prepared to share your group descriptions with the whole class.